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GLMM and GEE for Categorical Repeated Measures – Linking SAS and R

Miriam Amor, Veramed, Madrid, Spain

ABSTRACT

Since R started being used in clinical trials, hybrid SAS and R programming has become a reality. Collaborative initiatives, such as like PHUSE's CAMIS working group, are leading the way in facilitating the alignment between different languages.

In statistics, repeated measures are frequently analysed, with MMRM models being well-known models to analyse continuous variables. However, when the outcome is categorical, different methodologies are needed. Statistical methods that rely on a link function such as GEE and GLMM (using Laplace approximation, PQL or GHQ) become key alternatives. While SAS provides procedures like GENMOD and GLIMMIX, R offers functions as glmer, glmmPQL and geeglm, and the combination of both is the perfect toolkit to analyse categorical variables with repeated measures.

This presentation aims to show how to use R and SAS to program different models when the outcome is a categorical variable (binomial and multinomial) with repeated measures.

INTRODUCTION

The use of open-source software in the industry is a present reality. R has gained significant traction and is often routinely used alongside SAS. As a result, modern programmers are expected to understand the nuances and potential discrepancies in statistical outputs across these tools, especially in statistical models.

To support this transition, the PHUSE CAMIS (Comparing Analysis Method Implementations in Software) initiative has developed a repository that documents and compares statistical methodologies implemented in SAS, R, and Pvthon.

One area where these differences are particularly relevant is the analysis of longitudinal data. While Mixed Models for Repeated Measures (MMRM) are widely known for continuous outcomes, categorical outcomes require alternative approaches such as Generalized Linear Mixed Models (GLMM) and Generalized Estimating Equations (GEE).

This paper explores how GLMM and GEE models for dichotomous outcomes under the frequentist framework are programmed in both SAS and R, highlighting key differences in syntax and default settings. Where feasible, strategies are proposed to align results across platforms to ensure consistency and reproducibility Additionally, this paper examines extensions of these models for outcomes with more than two categories are examined, including both nominal and ordinal outcomes.

GEE AND GLMM: AN OVERVIEW

Both GLMM and GEE methods extend the Generalized Linear Model (GLM) framework using link functions that relate the predictors to transformed outcome variable. For dichotomous response variables, the link functions is commonly the probit (in case of rare events complementary log-log may be preferable). For outcomes with more than two categories of response the cumulative link function is used in case of ordinal variables and generalized logit for nominal variables.

Though both methods are based on the link function, the modelling approach differs. Table 1 summarizes differences between the two models.

Table 1. Comparison of GEE and GLMM

	GEE	GLMM
Model Type	Marginal (Population-averaged effects)	Conditional (Subject-average effects)
Use case	Population-level inference	Individual-level inference
Random Effects	Not explicitly included	Explicitly Included
Correlation	Working correlation structure	Modelled via random effects
Estimation method	Quasi-likelihhod via iterative solving of estimating equations	Maximum Likelihood, based on approximations Gauss Hermite Quadrature - Laplace Penalized Quasi-Likelihood:

SAS PROCEDURES AND R FUNCTIONS FOR GEE AND GLMM: AN OVERVIEW

In SAS, PROC GEE or PROC GENMOD can be used to compute GEE models, while PROC GLIMMIX is commonly used to fit GLMMs, as it allows random effects.

In R, GEE models can be fitted using <code>gee::gee</code> or <code>geepack::geeglm</code>. For GLMMs, several packages offer functions with varying capabilities and optimization strategies. The most widely used include: <code>lme4::glmer,glmmTMB::glmmTMB,GLMMadaptive::mixed_model,</code> and <code>MASS::glmmPQL</code>. Table 2 summarizes which types of GLMMs can be fitted with each function.

Table 2. GLMM Model Types supported by SAS Procedures and R Functions

GLN	ИΜ	QHQ	LAPLACE	PQL
SAS	GLIMMIX	✓	✓	✓
	glmer	✓	✓	×
œ	glmTMB	×	✓	×
	mixed_model	✓	✓	X
	glmPQL	×	×	✓

Table 3 summarizes some key differences between SAS procedures and R functions, differences that are not limited to that list, as additional distinctions such as convergence criteria and optimization methods may exist.

Table 3. GLMM Model Types supported by SAS Procedures and R Functions

							Link F	unction	
Model	Software	Procedure/ Function	SE	Correlation Matrix	DF	Probit	Logit	Clogit	Glogit
5	SAS	GLIMMIX	Model-Based	NA	BW	√	√	√	✓
Ξ		glmer	Model-Based	NA	Inf.	✓	✓	✓	✓
GLMM	œ	glmTMB	Model-Based	NA	Inf.	✓	✓	×	×
	ш.	mixed_model	Model-Based	NA	Inf.	✓	✓	×	×
		glmPQL	Model-Based	NA	Residual	×	✓	×	×
	S	GENMOD	Sandwich	Independent	NA	✓	✓	✓	×
	SA	GEE	Sandwich	Independent	NA	✓	✓	✓	✓
GEE		gee	Sandwich	Independent	NA	✓	✓	×	×
ര	œ	geeglm	Sandwich	Independent	NA	✓	✓	X	×
	4	ordLORgee	Sandwich	Exchangeable	NA	×	×	✓	×
		nomLORgee	Sandwich	Exchangeable	NA	x	×	×	✓

DF: Degrees of Freedom. BW: Between-Within. LR: Likelihood-ratio test. Clogit: Cumulative Logit. Glogit: General Logit. NA: Not applicable.

Given the variety of available modelling approaches, it is not surprising that results may differ depending on the method selected.

This paper aims to provide a comparative overview of these functions and procedures for fitting GLMM models in R and SAS through a series of illustrative examples. Particular attention is given to identifying functions that yield matching results.

EXAMPLE DATA

A SAS dataset of clinical trial data comparing two treatments for a respiratory disorder available in "Gee Model for Binary Data" in the SAS/STAT Sample Program Library [1] is used to create these examples. These data are from Stokes, Davis, and Koch (2000) [2].

Patients in each of two centers are randomly assigned to receive active treatment or placebo. During treatment, respiratory status, represented by the variable *outcome* (coded here as 0=poor, 1=good), is determined for each of four visits. The variables *center*, *treatment*, *sex*, and *baseline* (baseline respiratory status) are classification variables with two levels. The variable *age* (age at time of entry into the study) is a continuous variable.

To uniquely identify subjects, a new variable USUBJID was created by concatenating SITE and ID. Variables TREATMENT, BASELINE, and VISIT were renamed to TRTP, BASE, and AVISITN. Two additional variables were generated using random values to simulate ordinal and nominal multi-level outcomes to assess the models with these outcomes (See Appendix 3).

GLMM and GEE models were fitted using treatment, visit and treatment by visit interaction as fixed effects, with random intercepts and subject-level effects.

In R, the variables OUTCOME, TRTP and AVISITN were converted to factors. Since the modeling functions use the first (alphabetically) level as the reference category, TRTP levels are ordered as 'P' (placebo) and 'A' (active), to ensure that placebo is used as the reference category in the models:

```
resp$trtp<-factor(resp$trtp, levels=c('P', 'A'))
resp$avisitn<-factor(resp$avisitn)
resp$outcome<-factor(resp$outcome)</pre>
```

GENERALIZED ESTIMATED EQUATIONS (GEE)

In SAS, GEE and GENMOD procedures can be used with identical syntax, while in R we found functions such as, gee::gee and geepack::geeglm are available, note that geepack::geeglm requires the response variable to be numeric (0/1), with 1 indicating the event of interest.

In both SAS and R, fitting a GEE model involves specifying the fixed effects, the binomial distribution for binary outcomes, the link function (specified as link=logit). The unique subject and the correlation structure are defined in repeated subject.../type=(or corr=)... statement in SAS, and through the id= and corstr= arguments in R. Both use an independent correlation matrix by default.

The Robust (Sandwich) S.E. is computed by default, while <code>gee::gee</code> also provides model-based (naïve) S.E. In SAS, model-based S.E. can be easily obtained by including the <code>modelese</code> option in the repeated statement.

Predicted probabilities and odds ratios (ORs) can be obtained in SAS using the LSMEANS statement:

- The ILINK option provides back-transformed predicted probabilities.
- The DIFF option, combined with either EXP or ODDSRATIO, computes ORs.
- CL computes confidence intervals.

In R, these estimates are not directly available from the model functions but can be derived using the well-known package <code>emmeans</code>. Table 4 summarizes the code used, the probabilities and ORs are computed for the treatment by visit interaction.

Table 4. GEE: SAS® and R code

```
proc gee data=resp;
    class trtp(ref="P") avisitn(ref='1') usubjid;
    model outcome(event='1') = trtp avisitn trtp*avisitn/ dist=bin link=logit;
    lsmeans trtp*avisitn/cl exp ilink oddsratio diff;
    repeated subject=usubjid/type=ind /*modelse [1]*/;
run;
```

[1] Option to display the model-based (naïve) S.E. along with the Sandwich S.E.

This example shows syntax with PROC GEE, and the same syntax can be used in PROC GENMOD. While the syntax is equivalent, minor differences in results may be found in the later decimal places. See Appendix 2 for details on comparison results across SAS procedures).

The estimated parameters obtained from the code above are presented in Table 5 SAS outputs have been processed for better visual comparison. Parameter estimates and SE match up to four decimal places, except for GEE results, where S.E. is rounded to three decimal places.

Table 5. GEE model: Estimated parameters. Results obtained with SAS® and R.

		;	SAS				R							
							geepack::geegl							
Parm	Level1	Level2	Estimate	StdErr	Z	ProbZ	(Intercept)	Estimate : -0.0351	5td.err Wald 0.2649 0.02	4 1 12				
Intercept			-0.0351	0.2649	-0.13	0.8946	trtpA avisitn2	0.8128 -0.4292	0.3950 4.23 0.2636 2.65					
trtp	A		0.8128	0.3950	2.06	0.0396	avisitn3 avisitn4	-0.1408 -0.2118	0.2983 0.22 0.2534 0.70	0.64				
avisitn	2		-0.4292	0.2636	-1.63	0.1034	trtpA:avisitn2	0.5165	0.4104 1.58	0.21				
avisitn	3		-0.1408	0.2983	-0.47	0.6370	trtpA:avisitn3 trtpA:avisitn4	0.3186 -0.1140	0.4284 0.55 0.3948 0.08					
avisitn	4		-0.2118	0.2534	-0.84	0.4034	gee::gee							
trtp*avisitn	A	2	0.5165	0.4104	1.26	0.2082	(Intercept) -0.0	351 0.	267 -0.131	0.265 -0.132				
trtp*avisitn	A	3	0.3186	0.4284	0.74	0.4570	avisitn2 -0.4	292 0.	399 2.039 383 -1.120	0.395 2.058 0.264 -1.628				
trtp*avisitn	A	4	-0.1140	0.3948	-0.29	0.7729	avisitn3 -0.1 avisitn4 -0.2	118 0.	379 -0.372 380 -0.558	0.298 -0.472 0.253 -0.836				
							trtpA:avisitn2 0.5 trtpA:avisitn3 0.3 trtpA:avisitn4 -0.1	186 0.	570 0.906 570 0.559 558 -0.204	0.410 1.258 0.428 0.744 0.395 -0.289				

In this example, the LSMEANS option in SAS was used to obtain probabilities and treatment comparisons by visit. In R, similar functionality is provided by the <code>emmeans</code> and contrast functions from the <code>emmeans</code> package. The code below shows how to compute these estimates in R using results from <code>geepack::geeglm</code>, offering a parallel approach to the SAS implementation:

```
#Get predicted probabilities for each treatment.

#Get predicted probabilities for each treatment, using the covariance matrix computed by the model

prob <- emmeans(model, ~ trtp*avisitn, data=resp, vcov.method=vcov(model),

type='response')

prob

#Get differences between treatments by visit (option "revpairwise" is used to compare A vs P)

diffs_mean<-emmeans(model, ~ trtp*avisitn, data=resp, vcov.method=vcov(model))

diffs <- contrast(diffs_mean, "revpairwise", simple="trtp")

diffs <- as.data.frame(diffs)
```

```
#Calculate CI (alpha=0.05)
alpha < -0.05
z crit <- qnorm(1 - alpha / 2)</pre>
diffs$low<-diffs$estimate - (z_crit*diffs$SE)</pre>
diffs$upp<-diffs$estimate + (z_crit*diffs$SE)</pre>
#Get OR applying exponential transformation;
or<-exp(diffs$estimate)</pre>
or low<-exp(diffs$low)
or upp<-exp(diffs$up)
#Get two-sided p-value
z <- diffs$estimate/diffs$SE</pre>
pvalue \leftarrow 2 * (1 - pnorm(z))
#Create a dataset with all the results
OR<-as.data.frame(cbind(diffs$avisitn, or,or_low, or_upp, z, round(pvalue,
digits=4)))
colnames(OR)<-c('avisit', 'OR', 'lower.CL', 'upper.CL', 'Z', 'p-value')</pre>
```

The results of estimated probabilities and ORs, obtained by using LSMEANS statement in SAS and the emmeans functions in R, are displayed below, showing identical results up to four decimal places.

Table 6. GEE model: Probabilities and ORs. Results obtained with SAS® and R.

				,	SAS								R			
	Effect	trtp	avisitn	Mu	StdErrMu	LowerMu	Upp	erMu	1	prob rtp avi	icitn	prob	SE	df	lower.cL	unner Cl
	trtp*avisitn	P	1	0.491	0.0662	0.365		0.619	P		131111	•	0.0662		0.365	0.619
ES	trtp*avisitn	A	1	0.685	0.0632	0.551		0.794	A				0.0632		0.550	0.795
ABILITIE	trtp*avisitn	P	2	0.386	0.0645	0.269		0.517	P	_			0.0645		0.269 0.569	0.518 0.810
BE	trtp*avisitn	A	2	0.704	0.0621	0.570		0.810	P	3			0.0660		0.332	0.586
	trtp*avisitn	P	3	0.456	0.0660	0.332		0.585	A				0.0610		0.589 0.316	0.825 0.569
PROB	trtp*avisitn	A	3	0.722	0.0610	0.589		0.825	Α				0.0663			0.731
Δ.	trtp*avisitn	P	4	0.439	0.0657	0.316		0.569								
	trtp*avisitn	A	4	0.611	0.0663	0.476		0.731								
	Effect t	rtp _tı	tp avisitn	OI	Lowe R OI		zValue	Probz	>	or avisit	01	2 lowe	er.CL u	pper	.CL Z	p-value
	trtp*avisitn A	P	1		2.254 1.03	9 4.889	2.06	0.0396	1		2.2		039		.89 2.06	•
OR	trtp*avisitn A	P	2	:	3.778 1.71	3 8.333	3.29	0.0010	2	2	3.78	3 1	.713	8	3.33 3.29	0.0010
	trtp*avisitn A	P	3		3.100 1.40	5 6.840	2.80	0.0051	3	3	3.10) 1	.405	6	.84 2.80	0.0051
	trtp*avisitn A	P	4		2.011 0.94	4 4.288	1.81	0.0704	4	4	2.0	L (. 944	4	.29 1.81	0.0704

GENERALIZED LINEAR MIXED MODEL (GLMM) - GHQ APROXIMATION

Likelihood approximation using GHQ is based on an integral split in a given number of points.

GLMM with GHQ approximation can be fitted using the GLIMMIX procedure in SAS, or the lme4::glmer and $GLMMadaptative::mixed_model R$ functions. The syntax is similar to GEE models, with the difference that GLMM incorporates intercept and subject as random effects (i.e.: random baseline and slope for each subject).

In GLMMs, intra-subject correlation is estimated through the random effects. In SAS the random effects are specified with the RANDOM statement, while the TYPE= option statement can be used to specify the covariance structure of G (variance matrix of the random effects). Variance Components by default (type=VC) is the default option. In R, the equivalent specification is 1|usubjid, where 1 denotes the random intercept.

Unlike in GEE models discussed in previous section, which computed the robust Sandwich S.E. by default, the GLIMMIX procedure in SAS and the R functions used in this section display the model-based S.E. (also called naïve S.E. in R) by default.

In this example, a GHQ proximation with 5 quadrature points is used. The number of points is specified using method=quad(qpoints=n) in SAS, and nAGQ=n in R. In SAS, the option solution needs to be added to display the

estimated parameters. Since degrees of freedom (df) by default differ between software (SAS uses between-within, while R uses Infinite), the option ddfm=none is included in SAS to align with R. The corresponding syntax is shown in Table 7.

Table 7. GLMM - GHQ approximation (5 points): SAS® and R code.

```
proc glimmix data=resp method=quad(qpoints=5);
  class trtp(ref="P") avisitn(ref='1') usubjid;
  model outcome=trtp avisitn trtp*avisitn/dist=bin link=logit solution
ddfm=none/*1*/;
  random intercept /subject=usubjid /*type=vc [2]*/;
  run;
  model <- glmer(outcome ~ trtp + avisitn + trtp*avisitn + (1 | usubjid),
                 data = resp,
                 family = binomial(link ="logit"),
                 nAGQ=5)
2
  model <- mixed model(fixed = outcome ~ trtp + avisitn + trtp*avisitn,</pre>
               random = \sim 1 | usubjid,
               data = resp,
               family = binomial(link = "logit"),
               nAGQ=5)
```

- [1] Option to modify df (Between-within by default), set to none to get same results as with R.
- [2] Covariance structure of G (residuals covariance matrix). Variance components (VC) by default.

The results obtained are shown in Table 8. The main general difference between SAS and R lies in how p-values are calculated: SAS uses the t- distribution, while R relies on the standard normal (Z) distribution. In R, the results from the glmer function closely align those from SAS, with differences found in the 5^{th} decimal (eg: -0.87191 vs. -- 0.87199 in parameter estimation for AVISITN=2). However, the results produced by the mixed_model function in R show slight deviations when compared to both GLIMMIX and glmer, suggesting differences in the model implementation or estimation

Table 8. GLMM - GHQ approximation (5 points): Estimated parameters. Results obtained with SAS® and R. Model Based S.E. and df=Inf.

			SAS					R				
Effect	trtp	avisitn	Estimate	StdErr	DF	tValue	Probt	glmer		Std. Error		
Intercept		_	-0.06170	.5274	I	-0.12	0.9069	(Intercept) trtpA	-0.0617 1.5750	0.5274 0.7835	-0.12 2.01	0.907 0.044
trtp	A	_	1.57500	.7835	I	2.01	0.0444	avisitn2 avisitn3	-0.8720 -0.2873	0.5476 0.5369	-1.59 -0.54	0.111 0.593
avisitn		2	-0.87191	.5476	I	-1.59	0.1113	avisitn4	-0.4318	0.5386		0.423
avisitn		3	-0.28727	.5369	I	-0.54	0.5926		1.0440 0.6362	0.8036 0.8000		0.426
avisitn		4	-0.43174	.5386	I	-0.80	0.4228	trtpA:avisitn4	-0.2200	0.7866	-0.28	0.780
trtp*avisitn	A	2	1.04384	.8036	I	1.30	0.1939	mixed_model				
trtp*avisitn	A	3	0.63608	.8000	I	0.80	0.4265	_		e Std.Err		•
trtp*avisitn	A	4	-0.22001	.7866	I	-0.28	0.7797	(Intercept) trtpA	-0.07 1.63		-0.131 1.878	0.90 0.06
								avisitn2 avisitn3 avisitn4 trtpA:avisitn trtpA:avisitn trtpA:avisitn	3 0.63	9 0.537 3 0.539 6 0.803 7 0.800	-1.597 -0.538 -0.804 1.303 0.797 -0.278	0.11 0.59 0.42 0.19 0.43 0.78

SANDWICH S.E. AND D.F.

However, Li, P., & Redden, D. T. (2015) [4], recommended using the Between-Within denominator degrees of freedom approximation method when using GLMMs in cluster randomized trials with binary outcomes and a small number of heteregoneous clusters.

Additionally, the U.S. Food and Drug Administration (FDA) advises sponsors to consider using of robust standard error method such as the Huber-White "sandwich" standard error, particularly when the model does not include

treatment by covariate interactions [5].

Therefore, estimatations may need to incorporte the between-within df and/or the Sandwich robust S.E.. In SAS, df method can be specified using the dffm option in the model statment (with Between-Within being the default, or ddfm=BW). The Sandwich S.E. is easily obtained by adding the empirical option in the proc statment.

In R, lme4::glmer function does not have these options, and suplementary functions such as merDeriv::sandwich can be used to obtain Sandwich S.E. For the df, the function parameters::dof_betwithin can be used, though it implements a heuristic based on the between-within approach and does not return exactly the same results as shown in Li and Redden 2015 [4], but similar results are obtained [8].

Table 9. displays the R code to obtain the Sandwich S.E. A macro called <code>new_model</code> is created (See code in Appendix 1), to re-calculate p-values using the Sandwich S.E. and the specified df. These p-values are based in the t-distribution to align with SAS calculations.

Table 9. GLMM - GHQ approximation (5 points): SAS® and R code

```
proc glimmix data=resp method=quad(qpoints=5) empirical;
class trtp(ref="P") avisitn(ref='1') usubjid;
model outcome=trtp avisitn trtp*avisitn/dist=bin link=logit solution
ddfm=betwithin /*[1]*/;
random intercept /subject=usubjid /*type=vc*/; /*[2]*/;
 model <- glmer(outcome ~ trtp + avisitn + trtp*avisitn + (1 | usubjid),</pre>
               data = resp,
               family = binomial(link ="logit"),
               nAGQ=5)
#Get parameter estimation
est<-fixef(model)
#Get Sandwich covariance matrix
library(merDeriv)
vcov<-sandwich(model)
#Get S.E. (the diagonal from the covariance matrix), remove last value as it
corresponds to random effects
se sw0<- sgrt(diag(sandwich(model)))
se sw <-head(se sw0, -1)
#Re-calculate p-values using different df
new model(model, est fix=est, se=se sw , df=Inf)
new model (model, est fix=est, se=se sw , df=dof betwithin (model))
```

- [1] Option to set up df (Between-within by default), set to none to get same results as with R.
- [2] Covariance structure of G (residuals covariance matrix). Variance components (VC) by default.

The results (displayed in Table 10) are largely consistent when using infinite df, with only a minor difference in the estimated S.E. for the intercept (-0.8719 SAS vs. -0.8720 in R), attributable to differences in rounding (R rounds to the even digit (i.e. both 1.5 and 2.5 round to 2), while SAS uses "conventional" rounding rules (i.e 1.5 is rounded to 2 and 2.5 to 3 [6]) However, the estimated df differ between the two software (estimation in R is an aproximate version of the between-within method). This discrepancy in the df leads to small variations in the p-value computations across software.

Table 10. GLMM - GHQ approximation (5 points): Estimated parameters. Results obtained with SAS® and R. Sandwich S.F.

				SAS							R				
	Effect	trtp	avisitn	Estimate	StdErr	DF	tValue	Probt		Estimate	Std.				P-value
	Intercept		_	-0.0617	.5345	I	-0.1154	0.9081	(Intercept) trtpA	-0.0617 1.5750				-0.115 1.990	0.9087 0.0466
	trtp	A	_	1.5750	.7868	I	2.0019	0.0453	avisitn2 avisitn3	-0.8720 -0.2873				-1.617 -0.471	0.1058 0.6378
J-	avisitn		2	-0.8719	.5391	I	-1.6173	0.1058	avisitn4	-0.4318		0.516	Inf	-0.836	0.4031
DF: Inf	avisitn		3	-0.2873	.6102	I	-0.4708	0.6378	trtpA:avisitn2 trtpA:avisitn3	1.0440 0.6362		0.822		1.270 0.741	0.2041 0.4589
□	avisitn		4	-0.4317	.5164	I	-0.8361	0.4031	trtpA:avisitn4	-0.2200		0.794	Inf	-0.277	0.7817
	trtp*avisitn	A	2	1.0438	.8219	I	1.2700	0.2041							
	trtp*avisitn	A	3	0.6361	.8588	I	0.7406	0.4589							
	trtp*avisitn	A	4	-0.2200	.7939	I	-0.2771	0.7817							
										Estimate	C+d	Error	д£	+value	P-value
	Effect	trtp	avisitn	Estimate	StdErr	DF	tValue	Probt	(Intercept)	-0.0617	Stu.			-0.115	0.9088
	Intercept		_	-0.0617	.5345	109	-0.1154	0.9083	trtpA avisitn2	1.5750 -0.8720				1.990 -1.617	0.0474 0.1068
hin	Intercept trtp	A		-0.0617 1.5750					avisitn2 avisitn3	-0.8720 -0.2873		0.539 0.610	324 324	-1.617 -0.471	0.1068 0.6381
Within	-	A		1.5750	.7868	109	2.0019	0.0478	avisitn2 avisitn3 avisitn4 trtpA:avisitn2	-0.8720 -0.2873 -0.4318 1.0440		0.539 0.610 0.516 0.822	324 324 324 324	-1.617 -0.471 -0.836 1.270	0.1068 0.6381 0.4037 0.2050
en-Within	trtp	A	_	1.5750 -0.8719	.7868	109 327	2.0019	0.0478 0.1068	avisitn2 avisitn3 avisitn4 trtpA:avisitn2 trtpA:avisitn3	-0.8720 -0.2873 -0.4318		0.539 0.610 0.516 0.822 0.859	324 324 324 324 324	-1.617 -0.471 -0.836	0.1068 0.6381 0.4037 0.2050
tween-Within	trtp avisitn	A	_ 2	1.5750 -0.8719 -0.2873	.7868 .5391	109 327 327	2.0019 -1.6173 -0.4708	0.0478 0.1068 0.6381	avisitn2 avisitn3 avisitn4 trtpA:avisitn2 trtpA:avisitn3 trtpA:avisitn4	-0.8720 -0.2873 -0.4318 1.0440 0.6362		0.539 0.610 0.516 0.822 0.859	324 324 324 324 324	-1.617 -0.471 -0.836 1.270 0.741	0.1068 0.6381 0.4037 0.2050 0.4594
Between-Within	trtp avisitn avisitn		2 3	1.5750 -0.8719 -0.2873	.7868 .5391 .6102 .5164	109 327 327 327	2.0019 -1.6173 -0.4708	0.0478 0.1068 0.6381 0.4037	avisitn2 avisitn3 avisitn4 trtpA:avisitn2 trtpA:avisitn3 trtpA:avisitn4	-0.8720 -0.2873 -0.4318 1.0440 0.6362		0.539 0.610 0.516 0.822 0.859	324 324 324 324 324	-1.617 -0.471 -0.836 1.270 0.741	0.1068 0.6381 0.4037 0.2050 0.4594
	trtp avisitn avisitn avisitn	A		1.5750 -0.8719 -0.2873 -0.4317 1.0438	.7868 .5391 .6102 .5164	109 327 327 327 327	2.0019 -1.6173 -0.4708 -0.8361 1.2700	0.0478 0.1068 0.6381 0.4037 0.2050	avisitn2 avisitn3 avisitn4 trtpA:avisitn2 trtpA:avisitn3 trtpA:avisitn4	-0.8720 -0.2873 -0.4318 1.0440 0.6362		0.539 0.610 0.516 0.822 0.859	324 324 324 324 324	-1.617 -0.471 -0.836 1.270 0.741	0.1068 0.6381 0.4037 0.2050 0.4594
DF: Between-Within	trtp avisitn avisitn avisitn trtp*avisitn	A A	2 3 4 2	1.5750 -0.8719 -0.2873 -0.4317 1.0438 0.6361	.7868 .5391 .6102 .5164 .8219	109 327 327 327 327 327	2.0019 -1.6173 -0.4708 -0.8361 1.2700 0.7406	0.0478 0.1068 0.6381 0.4037 0.2050 0.4594	avisitn2 avisitn3 avisitn4 trtpA:avisitn2 trtpA:avisitn3 trtpA:avisitn4	-0.8720 -0.2873 -0.4318 1.0440 0.6362		0.539 0.610 0.516 0.822 0.859	324 324 324 324 324	-1.617 -0.471 -0.836 1.270 0.741	0.1068 0.6381 0.4037 0.2050 0.4594
	trtp avisitn avisitn avisitn trtp*avisitn trtp*avisitn	A A	2 3 4 2 3	1.5750 -0.8719 -0.2873 -0.4317 1.0438 0.6361	.7868 .5391 .6102 .5164 .8219	109 327 327 327 327 327	2.0019 -1.6173 -0.4708 -0.8361 1.2700 0.7406	0.0478 0.1068 0.6381 0.4037 0.2050 0.4594	avisitn2 avisitn3 avisitn4 trtpA:avisitn2 trtpA:avisitn3 trtpA:avisitn4	-0.8720 -0.2873 -0.4318 1.0440 0.6362		0.539 0.610 0.516 0.822 0.859	324 324 324 324 324	-1.617 -0.471 -0.836 1.270 0.741	0.1068 0.6381 0.4037 0.2050 0.4594

To obtain probabilities and OR, the same code as described in GEE models can be used. To compute results using the Sandwich S.E., the sandwich function can be used in the corresponding argument in emmeans:

```
emmeans (model, ~ trtp*avisitn, data=resp, vcov.method=sandwich (model),...)
```

GENERALIZED LINEAR MIXED MODEL (GLMM) - LAPLACE APROXIMATION

Laplace is a particular GHQ where only one point is used. In SAS, it can be obtained in the method statement using either method=quad(qpoints=1) or method=laplace. Both approaches return similar results with slight differences in later decimal places (See Appendix 2 for comparison between approaches).

Table 11 presents the code used to fit the model using Laplace approximation. In R, two functions are highlighted: lme4::glmer and glmmTMB::glmmTMB. An initial attempt using glmer resulted in a convergence warning:

```
In checkConv(attr(opt, "derivs"), opt$par, ctrl = control$checkConv, : Model failed to converge with max|grad| = 0.00442931 (tol = 0.002, component 1).
```

This warning can be addressed by switching the optimizer to "bobyqa" and extending the maximum number of iterations. These adjustments not only suppress the warning but provides closer results to SAS [6].

The glmmTMB::glmmTMB function is also a viable alternative. Recent versions of the package R (>= 3.6.0) support the computation of Sandwich S.E. using the using clubSandwich::vcovHC function [7].

```
Table 11. GLMM - Laplace: SAS® and R code. Sandwich S.E.
```

```
proc glimmix data=resp method=laplace empirical;
class trtp(ref="A") avisitn(ref='1') usubjid;
model outcome=trtp avisitn trtp*avisitn/dist=bin link=logit solution ddfm=none;
random intercept /subject=usubjid /*type=vc*/;
run;
```

```
library(lme4)
model <- glmer(formula =outcome ~ trtp + avisitn + trtp*avisitn + (1 | usubjid),
               data = resp,
                family = binomial(link ="logit"),
                nAGQ=1,
                control=glmerControl(optimizer="bobyqa",
                        optCtrl=list(maxfun=100000)))
#Get estimator
est<-fixef(model)
library(merDeriv)
#Get Sandwich covariance matrix
vcov<-sandwich(model)
#Get S.E. (the diagonal from the covariance matrix), remove last value as it
corresponds to random effects
se sw0<- sqrt(diag(sandwich(model)))
se sw <-head(se sw0, -1)
#Re-calculate p-values using different df
new model(model, est_fix=est1, se=se_sw , df=Inf)
new model(model, est fix=est1, se=se sw , df=dof betwithin(model))
model<- glmmTMB(outcome ~ trtp + avisitn + trtp*avisitn + (1 | usubjid),</pre>
                data = resp,
                family = binomial(link = "logit"))
#Get estimator
est<-fixef(model)$cond
#Get Sandwich S.E.
library(clubSandwich)
se sw<- sqrt(diag(vcovHC(model)))
#Re-calculate p-values using different df
new model(model, est_fix=est, se=se_sw , df=Inf)
new model(model, est fix=est, se=se sw , df=dof betwithin(model))
```

Results are shown in Table 12. While the <code>glmer</code> function provides results reasonably close to SAS, the <code>glmmTMB</code> yields results that align more closely with SAS.

Table 12. GLMM - Laplace: Estimated parameters. Results obtained with SAS® and R. Sandwich S.E.

	Df: Inf.		•						Df: Between-Within							
	Effect	trtp	avisitn	Estimate	StdErr	DF	tValue	Probt	Effect	trtp	avisitn	Estimate	StdErr	DF	tValue	Probt
	Intercept		_	-0.0847	.5452	I	-0.1554	0.8765	Intercept		_	-0.0847	.5452	109	-0.1554	0.8768
	trtp	A	_	1.6517	.8182	I	2.0188	0.0435	trtp	A	_	1.6517	.8182	327	2.0188	0.0443
	avisitn		2	-0.8576	.5302	I	-1.6175	0.1058	avisitn		2	-0.8576	.5302	327	-1.6175	0.1067
AS	avisitn		3	-0.2819	.5986	I	-0.4709	0.6377	avisitn		3	-0.2819	.5986	327	-0.4709	0.6380
Š	avisitn		4	-0.4236	.5069	I	-0.8357	0.4033	avisitn		4	-0.4236	.5069	327	-0.8357	0.4039
	trtp*avisitn	A	2	1.0248	.8026	I	1.2768	0.2017	trtp*avisitn	A	2	1.0248	.8026	327	1.2768	0.2026
	trtp*avisitn	A	3	0.6222	.8397	I	0.7410	0.4587	trtp*avisitn	A	3	0.6222	.8397	327	0.7410	0.4592
	trtp*avisitn	A	4	-0.2050	.7727	I	-0.2653	0.7908	trtp*avisitn	A	4	-0.2050	.7727	327	-0.2653	0.7910
R: glmer	(Intercept trtpA avisitn2 avisitn3 avisitn4 trtpA:avis trtpA:avis	sitn2 sitn3	-0.08 1.69 -0.89 -0.28 -0.42 1.02	348 518 576 319 236 247	Error 0.479 I 0.697 I 0.531 I 0.599 I 0.507 I 0.804 I 0.771 I	nf - nf nf - nf - nf nf	2.369 1.615 0.471 0.836 1.275 0.741	0.8596 0.0179 0.1062 0.6377 0.4030	(Intercep	sitni sitni	-0.0 1.6 -0.8 -0.2 -0.4 2 1.0 3 0.6	848 518 576 819 236 247 222	Error 0.479 3 0.697 3 0.531 3 0.599 3 0.507 3 0.804 3 0.840 3 0.771 3	324 - 324 - 324 - 324 - 324 - 324	2.369 1.615 0.471 0.836 1.275 0.741	-value 0.8597 0.0184 0.1072 0.6380 0.4037 0.2031 0.4594 0.7903

		Estimate Std.	Error	df tvalue	P-value		Estimate Std.				
	(Intercept)	-0.0847	0.545 I	nf -0.155	0.8765	(Intercept)	-0.0847	0.545	324	-0.155	0.8766
В	trtpA	1.6514		nf 2.018			1.6514				0.0444
MB	avisitn2	-0.8575	0.530 I	nf -1.617	0.1000	avisitn2	-0.8575				0.1068
15	avisitn3	-0.2819	0.599 I	nf -0.471	0.05/0		-0.2819				0.6381
15	avisitn4	-0.4236	0.507 I	nf -0.836	0.7033	avisitn4	-0.4236				0.4040
늘	trtpA:avisitn2	1.0247	0.803 I	nf 1.277	0.201/	trtpA:avisitn2		0.803	324	1.277	0.2026
900	trtpA:avisitn3	0.6221		nf 0.741		trtpA:avisitn3					0.4593
0.100	trtpA:avisitn4	-0.2050	0.773 I	nf -0.265	0.7908	trtpA:avisitn4	-0.2050	0.773	324	-0.265	0.7910

GENERALIZED LINEAR MIXED MODEL (GLMM) - PQL APROXIMATION

The PQL approach uses linear approximations instead of likelihood, making it less accurate for binary outcomes compared to the GHQ or Laplace methods described above. In SAS, this is implemented by default using the Residual Pseudo-Likelihood method (method=RSPL), which is a refinement of PQL which incorporates residual adjustments to better approximate the marginal likelihood, in the GLIMMIX procedure. In R the PQL computation can be obtained using the glmmPQL function form the MASS package.

Table 13 shows the syntax for both software, where df in PROC GLIMMIX are set to residual, for consistency and Table 14 the corresponding results.

Table 13. GLMM - PQL: SAS® and R code

Table 14. GLMM - PQL: Estimated parameters. Results obtained with SAS® and R.

			SAS					R								
Effect	trtp	avisitn	Estimate	StdErr	DF	tValue	Probt					_	_			
Intercept		_	-0.01934	0.3915	436	-0.05	0.9606	(Intercept)	Value 0.000	Std.Error 0.413		t-value 0.000	•			
trtp	A	_	1.0241	0.5756	436	1.78	0.0759	trtpA avisitn2	1.220 -0.815	0.603 0.394		2.024	0.0454			
avisitn		2	-0.6228	0.4583	436	-1.36	0.1749	avisitn3	-0.268	0.388	327	-0.691	0.4898			
avisitn		3	-0.2047	0.4527	436	-0.45	0.6514	avisitn4 trtpA:avisitn2	-0.403 0.976	0.389 0.580		-1.037 1.684	0.3006 0.0931			
avisitn		4	-0.3079	0.4537	436	-0.68	0.4977	trtpA:avisitn3 trtpA:avisitn4		0.578 0.568		1.029 -0.352				
trtp*avisitn	A	2	0.7466	0.6767	436	1.10	0.2705									
trtp*avisitn	A	3	0.4563	0.6762	436	0.67	0.5001									
trtp*avisitn	A	4	-0.1560	0.6630	436	-0.24	0.8141									
			I			I										

Results differ because of the different computation methods (PQL vs. RSPL) across software. Since <code>glmmPQL</code> is widely recognized as less reliable for binary outcomes, more robust approaches such as Laplace or GHQ—discussed in previous sections—are generally preferred. Consequently, further investigation using <code>glmmPQL</code> is not pursued.

CATEGORICAL OUTCOME WITH MORE THAN TWO CATEGORIES

Although less common than binary outcomes, endpoints with more than two categories may be the outcome of interest, which can be either ordinal or nominal.

In SAS, similar syntax used for GEE and GLMM models can be applied by specifying a multinomial distribution and selecting the appropriate link function. Models with cumulative link functions apply to ordinal data and generalized logit models are fit to nominal data [3]).

In R, the functions described earlier (e.g.: glmer, glmmTMB, geeglm, etc.) do not support multinomial outcomes, as these functions relay on the family function, which does not include multinomial option. Nevertheless, the multgee package [9] provides two functions for estimating GEE models with categorical outcomes.

Table 15 displays the syntax used to fit GEE models. In R, two different functions are available in the <code>multgee</code> package [9], depending on the type of outcome: <code>ordLoRgee</code> for ordinal variables and <code>nomLoRgee</code> for nominal variables.

By default, the correlation matrix structure in SAS is independence. In contrast, the default in R is exchangeable, specified as "category.exch" for ordLORgee and "time.exch" for nomLORgee. To ensure consistency with the SAS default, the R code shown sets the correlation structure to be independent.

Table 15. GEE - Categorical outcome with more than two categories: SAS® and R code

rable	15. (JEE – Categorical outcome with more than two categories: SAS® and R code
	Ordinal	<pre>proc gee data=resp; class trtp(ref="A") avisitn(ref='1') usubjid; model respord=trtp avisitn trtp*avisitn/ dist=multinomial link=cumlogit;</pre>
50	Ord	<pre>repeated subject=usubjid /corr=ind; run;</pre>
SAS	Nominal	<pre>proc gee data=resp; class trtp(ref="A") avisitn(ref='1') usubjid; model respnom(event='Liver')=trtp avisitn trtp*avisitn/ dist=multinomial link=glogit; repeated subject=usubjid /corr=ind; run;</pre>
	Ordinal	<pre>model <- ordLORgee(formula = respord ~ trtp + avisitn + trtp*avisitn,</pre>
В	Nominal	<pre>model <- nomLORgee(formula = respnom ~ trtp + avisitn + trtp*avisitn,</pre>

For GLMM, in SAS, similar modifications apply to the GLIMMIX procedure. One notable limitation is that the LSMEANS statement does not work as expected in GLIMMIX with the multinomial distribution. However, ORs can still be obtained by using the oddsratio option in the model statement. No R functions equivalent to SAS's GLIMMIX procedure have been identified for handling multinomial distributions in a frequentist framework.

CONCLUSION

SPECIFIC CONCLUSIONS

GEE and GLMM are effective for modelling categorical outcomes using appropriate link functions: *logit* for binary, *cumlogit* for ordinal, and *glogit* for nominal outcomes. A key distinction is that GLMMs account for intra-subject correlation through random effects, while GEEs use a working correlation matrix.

For GEE models, PROC GEE/GENMOD in SAS and gee/geepack in R yield comparable results, based on robust (sandwich) standard errors.

For GLMMs, Laplace and Gauss-Hermite Quadrature (GHQ) are preferred over Penalized Quasi-Likelihood (PQL). In SAS, PROC GLIMMIX allows specification of the approximation method with x points (e.g: method=quad(qpoints=x) or method=laplace). In R, glmer supports GHQ via nAGQ=x, while glmmTMB is recommended for Laplace approximation.

- Df handling differs: SAS defaults to between-within (ddfm=bw), while R assumes infinite df. To align results, ddfm=none can be specified in SAS or the function dof_between can be used in R, though it is an approximation and does not exactly match with SAS computation.
- Model-based standard errors are default in GLMMs, but robust SEs can be obtained using the empirical option in SAS, or packages like merDeriv and clubSandwich in R.
- Predicted probabilities and odds ratios (ORs) can be extracted via lsmeans and/or oddsratio in SAS, and using the emmeans package in R.
- Convergence criteria and optimization methods may be considered as well when discrepancies across methods are found.

GENERAL CONCLUSIONS

Differences between SAS and R, (and even between different functions/procedures within same software) can lead to different results. These differences do not imply that one tool is more reliable than another but rather reflect variations in default settings and computational methods. Carefully reviewing and aligning these defaults is essential for consistency and reproducibility.

To support this effort the CAMIS repository [1] documents known differences between statistical implementations in different software (such SAS, R and/or Phyton). It is a valuable resource for understanding and resolving discrepancies, promoting transparency and reproducibility across platforms.

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CONTACT INFORMATION

Your comments and questions are valued and encouraged. Contact the author at:

Author Name: Miriam Amor Company: Veramed GmBH

Address: Registered Office: 5th Floor Regal House, 70 London Road, Twickenham TW1 3QS,

Work Phone: NA

Email: Miriam.amor@veramed.co.uk Website: https://www.veramed.co.uk

APPENDIX 1: MACRO CODE TO CALCULATE P-VALUES

Macro parameters:

MODEL: object resulting from a function (e.g.: geeglm, glmer, glmmTMB, etc.)

EST FIX: vector with estimated parameters for the fixed effects

SE: vector with the estimated S.E. for the fixed effects.

DF: Degrees of freedom (e.g.: Inf. or dof between(model), to use infinite or between-within, respectively.

Macro for objects from lme4::glmer:

```
new_model <- function(model, est_fix , se , df) {

#Re-calculate (2-sided) p-values using estimated parameters and its SE

t <- est_fix /se
 pvalue <-2*pt(q=abs(t), df=df, lower.tail=FALSE)

#Combine results in a data frame and add row names
 new_model <- round(cbind(est_fix, se, df, t, pvalue), digits=4)
  colnames(new_model) <- c("Estimate", "Std. Error", "df", "tvalue", "P-value")
  rownames(new_model) <- rownames(summary(model)$coefficients)
  new_model
}</pre>
```

Macro for objects from glmmTMB::glmmTMB:

```
new_model <- function(model, est_fix , se , df) {

#Re-calculate (2-sided) p-values using estimated parameters and its SE

t <- est_fix /se
   pvalue <-2*pt(q=abs(t), df=df, lower.tail=FALSE)

#Combine results in a data frame and add row names
   new_model <- round(cbind(est_fix, se, df, t, pvalue), digits=4)
   colnames(new_model) <- c("Estimate", "Std. Error", "df", "tvalue", "P-value")
   rownames(new_model) <- rownames(summary(model)$coefficients$cond)
   new_model
}</pre>
```

APPENDIX 2: COMPARISON BETWEEN SAS PROCEDURES RESULTS

GEE: PROC GEE VS PROC GENMOD

PROC GEE and PROC GENMOD share identical syntax. Results are compared, and, while exact match is not possible due to differences in least significant decimal places, a clean comparison accurate up to the 9th decimal place can be obtained using criterion in the PROC COMPARE.

```
proc gee data=resp;
  class trtp(ref="P") avisitn(ref='1') usubjid;
  model outcome(event='1')= trtp avisitn trtp*avisitn/ dist=bin link=logit;
  repeated subject=usubjid /corr=ind
  ods output  GEEEmpPEst=GEEEmpPEst_gee;
  run;

proc genmod data=resp;
  class trtp(ref="P") avisitn(ref='1') usubjid;
  model outcome(event='1')= trtp avisitn trtp*avisitn/ dist=bin link=logit;
  repeated subject=usubjid /corr=ind;
  run;

proc compare base=GEEEmpPEst_gee compare=GEEEmpPEst_genmod criterion=0.000000000001;
  run
```

```
The COMPARE Procedure
    Comparison of WORK.GEEEMPPEST_GEE with WORK.GEEEMPPEST_GENMOD
               (Method=RELATIVE(1), Criterion=1.0E-14)
                         Data Set Summary
Dataset
                                                Modified NVar
                               Created
                                                                 NObs
WORK.GEEEMPPEST GEE
                       9
                                                                   15
WORK.GEEEMPPEST GENMOD 01SEP25:08:31:07 01SEP25:08:31:07
                                                            9
                                                                   15
                         Variables Summary
                Number of Variables in Common: 9.
                         Observation Summary
                    Observation
                                    Base Compare
                   First Obs
                                       1
                   Last Obs
                                      15
                                               15
  Number of Observations in Common: 15.
  Total Number of Observations Read from WORK.GEEEMPPEST GEE: 15.
  Total Number of Observations Read from WORK.GEEEMPPEST GENMOD: 15.
  Number of Observations with Some Compared Variables Unequal: 0.
  Number of Observations with All Compared Variables Equal: 15.
                      Values Comparison Summary
   Number of Variables Compared with All Observations Equal: 9.
   Number of Variables Compared with Some Observations Unequal: 0.
   Total Number of Values which Compare Unequal: 0.
   Total Number of Values not EXACTLY Equal: 47.
   Maximum Difference Criterion Value: 2.1539E-15.
```

GLMM - LAPLACE: GLIMMIX with method=Laplace vs.method=quad(qpoints=1)

As Laplace is a special case of GHQ approximation using only 1 point, it can be specified in SAS using either method=laplace or method=quad(qpoints=1). Although both options return similar results, discrepancies may occur beyond the 8th decimal place may be found. Results would match using the criterion option in the proc compare.

```
The COMPARE Procedure
                  Comparison of WORK.PARAMETERLAPLACE with WORK.PARAMETERGHQ 1P
                         (Method=RELATIVE(0.00000222), Criterion=1.0E-08)
                                        Data Set Summary
                               Created
                                                Modified NVar NObs Label
Dataset
WORK.PARAMETERLAPLACE 01SEP25:08:19:28 01SEP25:08:19:28 8 15 Solutions for Fixed Effects
WORK.PARAMETERGHQ_1P 01SEP25:08:19:28 01SEP25:08:19:28 11 15 Solutions for Fixed Effects
                                        Variables Summary
                              Number of Variables in Common: 8.
                              Number of Variables in WORK.PARAMETERGHQ_1P but not in WORK.PARAMETERLAPLACE: 3.
                                       Observation Summary
                                  Observation
                                                  Base Compare
                                  First Obs
                                  Last Obs
                                                     15
                                                              15
                Number of Observations in Common: 15.
                Total Number of Observations Read from WORK.PARAMETERLAPLACE: 15.
                Total Number of Observations Read from WORK.PARAMETERGHQ 1P: 15.
                Number of Observations with Some Compared Variables Unequal: 0.
                Number of Observations with All Compared Variables Equal: 15.
                                    Values Comparison Summary
                 Number of Variables Compared with All Observations Equal: 8.
                 Number of Variables Compared with Some Observations Unequal: 0.
                 Total Number of Values which Compare Unequal: 0.
                 Total Number of Values not EXACTLY Equal: 32.
                 Maximum Difference Criterion Value: 2.2958E-09.
```

```
proc glimmix data=resp method=laplace empirical;
class trtp(ref="P") avisitn(ref='1') usubjid;
model outcome=trtp avisitn trtp*avisitn/ dist=bin link=logit solution ddfm=none;
random intercept /subject=usubjid;
ods output ParameterEstimates=ParameterLaplace;
run;

proc glimmix data=resp method=quad(qpoints=1) empirical;
class trtp(ref="P") avisitn(ref='1') usubjid;
model outcome=trtp avisitn trtp*avisitn / dist=bin link=logit solution cl ddfm=none;
random intercept /subject=usubjid;
ods output ParameterEstimates=ParameterGHQ_1p;
run;

proc compare base=ParameterLaplace compare=ParameterGHQ_1p criterion=0.00000001;
run;
```

APPENDIX 3: DERIVATION OF MULTI-LEVEL VARIABLES (ORDINAL AND NOMINAL)

```
proc format;
  value respmulti
  1='Liver'
  2='Lung'
  3='Bone';
  run;

data resp;
  set resp1;
  call streaminit(1234);
  respord = rand("integer", 1, 3); *Ordinal;
  respnom = put(respord, respmulti.); *Nominal;
  run;
```