

Determination of Body Sway Area by Fourier Analysis of its Contour

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Abstract

Posturography is used to assess the steadiness of the human body by measuring the movements of the centre of pressure (COP) of a standing subject on a force platform (stabilometry). This paper presents a new method of calculating the area of the centre of pressure (COP) trajectory (sway area). The outline of COP area is determined by detecting points that are furthest from the centre in a given angular interval. A Fourier series is fitted to this outline so that the points inside and outside are weighted differently. The SAS[®] procedure PROC SPECTRA is used to calculate the Fourier coefficients. With Fourier coefficients a series of sine and cosine waves that describe an approximation of the body sway outline is overlaid. Fourier analysis of the sway area contour is highly suitable for data interpretation. It gives not only the value of the sway area but also some information about its shape.

INTRODUCTION

Posturography is a general term that covers all techniques used to quantify postural control in upright stance under in either static or dynamic conditions. I will only concentrate on static posturography. Static posturography is carried out by placing the patient in a standing posture on a fixed-instrumented platform (force platform) connected to sensitive detectors (force and movement transducers), that are able to detect tiny oscillations of the body.

Posturography, also called test of balance, is a non-invasive specialized clinical assessment technique used to quantify the central nervous systems (sensory/motor) involved in the control of posture and balance.

The ability to perform routine activities of daily life requires stable control of posture and balance.

A balance disorder is a disturbance that causes an individual to feel unsteady, for example when standing or walking. It may be accompanied by feelings of dizziness, or having a sensation of movement, spinning, or floating. Balance is the result of several body systems working together: the visual system (eyes), vestibular system (ears) and proprioception (the body's sense of where it is in space). Degeneration or loss of function in any of these systems can lead to balance deficits [1].

The assessment of posture has been studied by measuring center of pressure (COP). The COP is an indirect measure of postural sway. The trajectory of the COP is monitored and can be displayed over a time interval. The sway area that is traced by the trajectory is an effective parameter for measuring postural sway.

In this paper I have used an efficient method of computation of body sway contour (Fig. 1a-1c) of a given sway path. This method was already introduced in [2] and compared with the PCA method. The sway path is reduced to an outline which keeps only points with a maximum distance to the origin based on a decomposition of the area into sectors (partitioning of a full revolution into a chosen number of intervals). The outline (Fig. 1b) is the basis of the resulting Fourier transformation decomposition.

I will analyse the area of the center of pressure (COP) by depicting of the area contour and then its description in terms of Fourier coefficients. The outline will be described by a series of Fourier coefficients (Fig. 2).

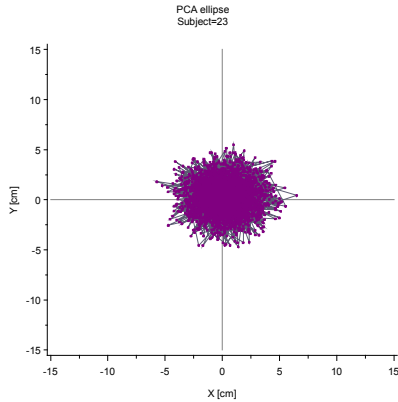


Fig. 1a: Posturography, sway path with simulation data

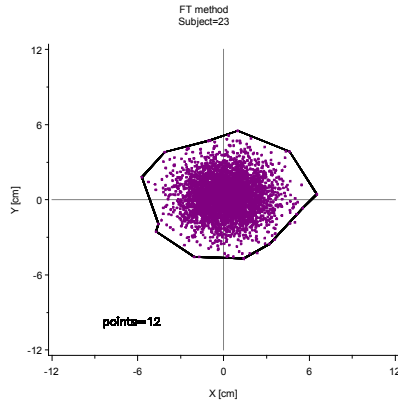


Fig. 1b: Contour reduced to concave polygon

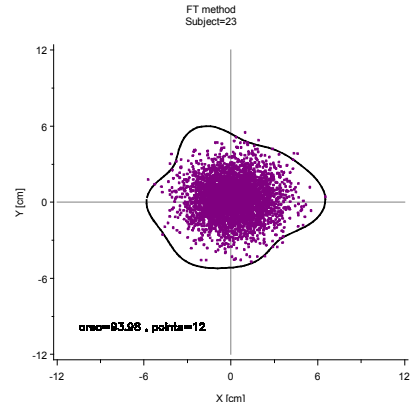


Fig. 1c: Resulting contour calculated by FT

At the end of the paper I will compare the Fourier method with two other effective methods - the Convex hull and PCA (principal component analysis) to calculate the area and contour of a given set of points.

The following equation describes the sway area outline expressed in polar coordinates $R(\Phi)$, where R is the radius – it is the distance from the origin of the coordinate system to the outline point given a polar angle Φ . The equation of the Fourier transform (FT) decomposition of the series given an angle Φ is.

$$R(\Phi) = \frac{R_0}{2} + \sum_{m=2}^{m_{\max}} [A_m \cos(m\Phi) + B_m \sin(m\Phi)] \quad (1)$$

Where m is the number of frequencies in the Fourier decomposition.

$R(\Phi)$ describes the function of the radius R for a given angle Φ . A_m and B_m are the Fourier coefficients which build the weights of the decomposition into sine and cosine waves.

SAS PROCEDURE PROC SPECTRA

Proc SPECTRA performs spectral and cross-spectral analysis of time series. One can use the procedure to analyse the data for periodic or cyclic patterns. Proc SPECTRA produces estimates of the spectral and cross-spectral densities of a multivariate time series. Estimates of the spectral densities are calculated using a Fourier transformation. It returns periodograms and cross-periodograms. I used SAS Proc SPECTRA to calculate the Fourier coefficients [4].

```
proc spectra data=ft out=fouriercoeff coef;
    var r ;
    by usubjid;
run;
```

Proc SPECTRA uses the finite Fourier transform to decompose data series into a sum of sine and cosine waves of different amplitudes and wavelengths. In the VAR statement one specifies the variables to be analysed. Fourier coefficients are calculated and written to the OUT=data set with the COEF statement.

The Fourier coefficients A_m and B_m are given below in Fig. 2 as example as COS_01 and SIN_01 respectively.

FREQ Frequency from 0 to PI	PERIOD Period	COS_01 Cosine Transform of r	SIN_01 Sine Transform of r	P_01 Periodogram of r	i
0	.	17.426442167	0	1518.4044331	1
0.6283185307	10	-0.655461034	-0.19103331	2.3306144627	2
1.2566370614	5	0.3359201004	0.1221890468	0.6388623851	3
1.8849555922	3.3333333333	-0.166455607	-0.388021996	0.8913426937	4
2.5132741229	2.5	0.1449707908	0.1991130957	0.3033127753	5
3.1415926536	2	0.0967784117	0	0.0468303048	6

Fig. 2 Fourier coefficients A_m (COS_01) and B_m (SIN_01) calculated with Proc SPECTRA

The area of the contour can be calculated from the Fourier coefficients by use of the following equation:

$$A = \int_0^{2\pi} R(\Phi) dR d\Phi = \pi \frac{R_0^2}{2} + \pi \sum_{m=2}^{m_{\max}} [A_m^2 + B_m^2] \quad (2)$$

CACULATION OF THE SWAY AREA OUTLINE

Firstly we need to determine a reduced point set that can be used as an outline of the sway path and which will be inserted in the Fourier transformation. To calculate the sway area outline, all data n points (x_i, y_i) are converted into polar coordinates with radius R_i and polar angle Φ_i . The full revolution (360°) is then divided into chosen number of intervals m ($360/m$ sectors). For good results at least 30 to 60 points are usually suitable. The more points used, the more details of the shape are shown. For each angular interval the point with the largest distance R_i' is deemed the representative point on the outline approximation (Fig. 1b). These points build up a concave polygon that will be used as the starting point of the Fourier transformation. The next step is the determination of the smooth sway area with Fourier transformation.

ALGORITHM FOR FOURIER TRANSFORM DECOMPOSITION

I developed a macro to calculate the Fourier transform decomposition of the contour. A Fourier transform decomposition is calculated for the reduced point set. Proc SPECTRA returns the $(n/2)+1$ (if n is even) or $(n+1)/2$ Fourier coefficients if n is odd which compose the Fourier series. The series is a sum of sine and cosine terms. Each term is multiplied by a specific Fourier coefficient. The more points are included, the closer the approximation of the contour is to the primary point set.

The following SAS code is the main part of the SAS macro %fouriertransform. It shows the calculation of the radius R_i by Fourier transform decomposition for each angle Φ . The calculation is inside a loop from $-\pi$ to π . The resulting R are summarized for each Φ . It also calculates the area A , of the contour given by equation (2):

```
data ft;
  set ft;
  array r (&nmax);
  array b (&nmax);
  array a (&nmax);
  array area (&nmax);
  r_=0;
  area_=0;
  do phi=-&pi to &pi by &step_fourier ;
    r_=0;
    area_=0;
    do i=2 to &nmax by 1;
      r{i}=a{i}*cos(phi*i)+b{i}*sin(phi*i);
      area{i}=a{i}**2 + b{i}**2;
      area_=area_+area{i};
      r_=r_+r{i};
    end;
  end;
```

```

if i=&nmax then do;
    r_=r_+(a{1})/2;
    area_=&pi*area_+&pi*(a{1}/2)**2;
    output;
end;
end;
end;
run;

```

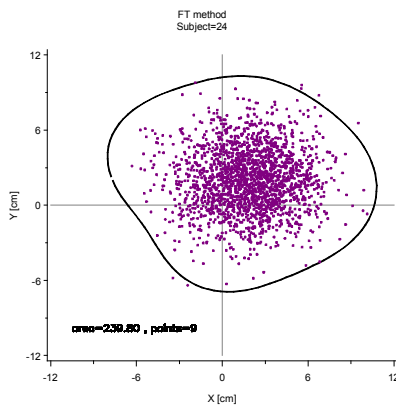
The sum of Fourier coefficients is calculated by equation (1).

The macro %fouriertransform is adjustable for the number of angles that divides the full angle into partitions and thus into certain number of points which constitute the first approximation of the outline. This approximation is then used as a starting set to calculate the Fourier transformation and the resulting smooth sway outline.

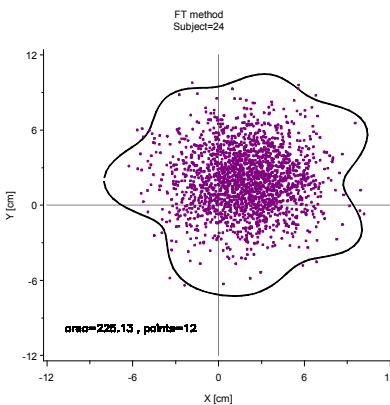
The following figure (Fig. 3) shows different approximations of the contour of a given point set. For each Fourier transformation a different number of points was used to calculate the Fourier coefficients. The more points are used the better is the approximation of the contour.

A minimum of 3 points are needed to calculate the Fourier coefficients, otherwise Proc SPECTRA returns an error. Calculating the coefficients with just 3 points generates a nearly perfect circle. Introducing more points in Proc SPECTRA results in more Fourier coefficients being calculated and the resulting outline shows more details of the given sway outline. In the example below I tried to include 9 to 90 points to be used to calculate the Fourier coefficients.

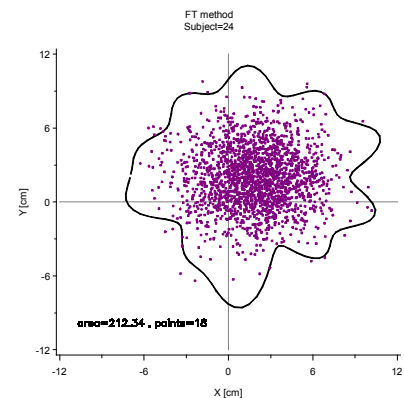
One can see the more details of the sway contour are calculated (more points) the smaller is the calculated area. With 3 points the area is maximum.



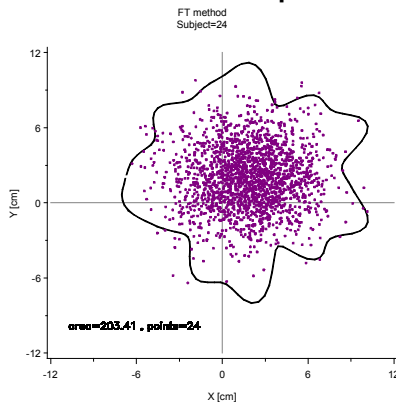
3a: FT with 9 contour points



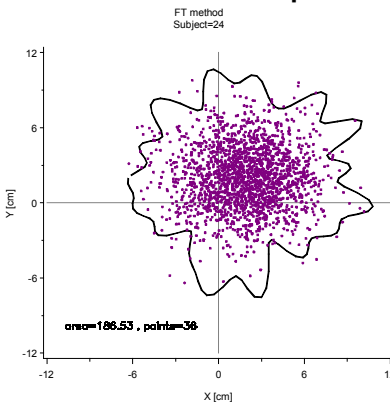
3b: FT with 12 contour points



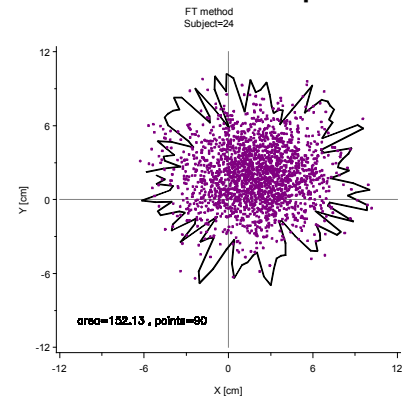
3c: FT with 18 contour points



3d: FT with 24 contour points



3e: FT with 36 contour points



3f: FT with 90 contour points

Fig. 3) Contour calculated with Fourier coefficients

COMPARISON OF FT OUTLINE CALCULATION WITH CONVEX HULL AND PCA METHOD

In [3] I previously discussed the approximation of the sway outline via the convex hull and PCA methods. Both methods are suitable and reliable to describe the sway outline. They were used to calculate the area of the given point set and compared with the FT method introduced in this paper.

The convex hull is a finite point set. A point q is 'visible' from p when the connection between p and q is completely in the polygon (Fig. 4a-b).

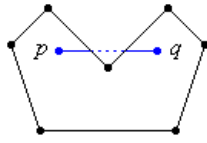
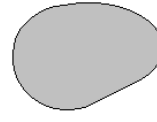
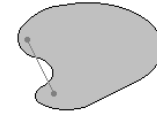


Fig. 4a) Point q is not visible from p



(a)



(b)

Fig. 4b) convex (a) and concave (b) point set

A polygon is convex if two points inside the polygon also include the connection between those two points. In Fig. 4b a convex (a) and a concave (b) polygon are shown. To calculate the convex hull I used triangulation of the given point set. Proc G3GRID determines a triangulation of the point set. The outer triangles describe the resulting convex hull (Fig. 5a). The convex hull of a point set is always unique.

In Fig. 5b we see a picture of the point set surrounded by a bounding box with an ellipse inside. This ellipse is calculated using the PCA method. The PCA is a mathematical procedure that uses an orthogonal transformation to convert a set of observations of possibly correlated variables into a set of values of uncorrelated variables called principal components. In [3] I used Proc PRINCOMP to derive the principal components. The resulting eigenvectors of the covariant matrix generate the vectors that form the ellipse with a specific main axis given as the main angle of the ellipse. The PCA method also returns a unique ellipse.

In Fig. 5a-c the results of the three methods are shown with the calculated sway outline and the area that is surrounded by the polygon or ellipse. The area calculated by the FT method depends on the number of points used for the calculation. The more points used, the smaller is the area. The resulting sway outline also shows more details if we use more points. For the convex hull and PCA method the sway outline is unique and only depends on the given point set.

To compare the methods I used simulation data.

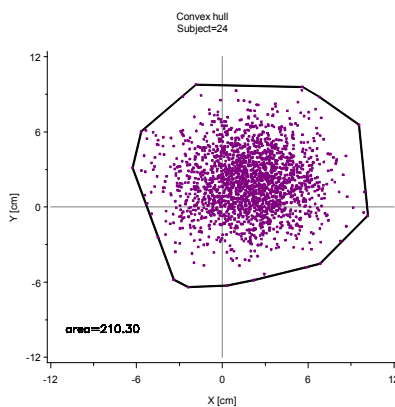


Fig. 5a) Convex hull (area=210.3)

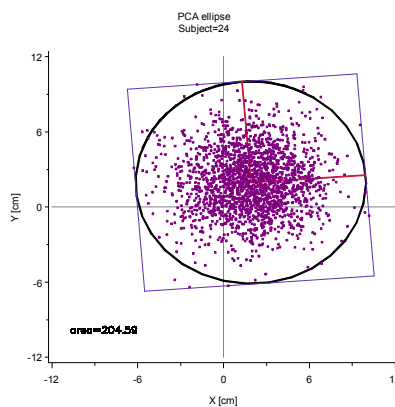


Fig. 5b) PCA method (area=204.6)

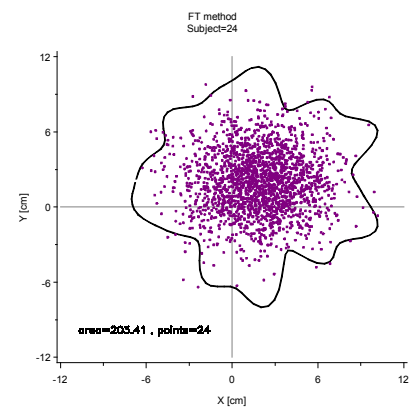


Fig. 5c) FT (area=203.4)

EFFECT OF OUTLIERS

The following figures (Fig. 6a-c) show the influence of outliers on the resulting curve. The convex hull (Fig 6a) always encloses all data points. Outliers have a large influence on the area of the convex hull. The PCA method is also not invariant against outliers (Fig. 6b) – it also tries to 'enclose' the data points and

the calculated ellipse becomes larger. In contrast the FT method is more robust against outliers. The influence of extreme values on the FT area is smaller than for the other two methods.

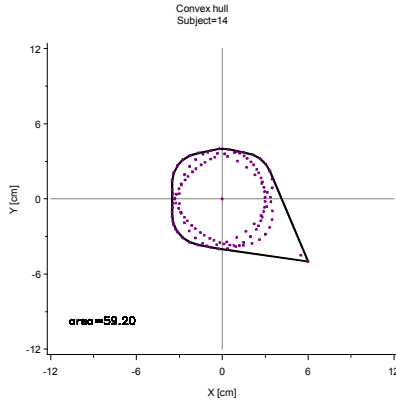


Fig. 6a) Convex hull (area=59.2)

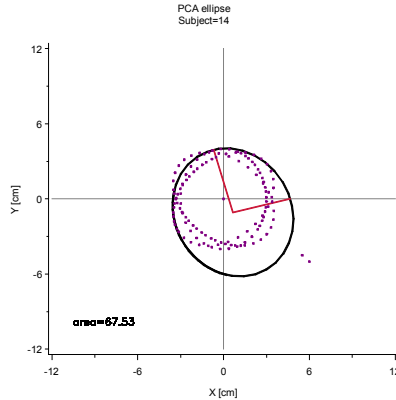


Fig. 6b) PCA method (area=67.5)

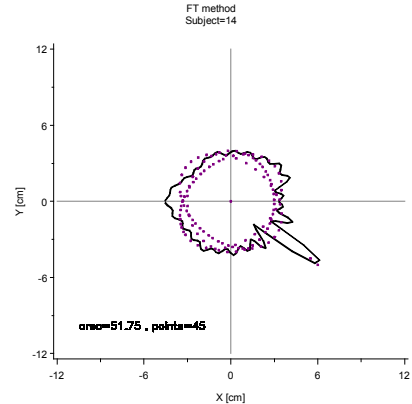


Fig. 6c) FT (area=51.8)

CONCLUSION

It was shown that the determination of the sway outline with Fourier analysis is suitable for data interpretation. The shape of the sway outline depends on the number of points included in the first approximation of the outline polygon. The outline reasonably describes the shape of the area. During comparison of the three methods it was shown that the area calculated by Fourier transformation is usually smaller, because it describes more details with concave embayments, whereas the convex hull and PCA methods always describe a convex course of the outline.

Convex hull always include all data points – PCA and FT method do not necessarily surround the point set. One advantage of the PCA method is that it also returns an angle of the main axis of movement. The thus determined ellipse always lies on this main axis. The FT method describes the actual outline with more details and can be adjusted based on the number of points used for calculation.

The FT method could be refined by adding a step to minimize the distance between the calculated points of the contour and the experimental points of the outline. This will result in a minimization problem of the square sum of the difference between experimental and calculated points. If the calculated point is closer to the origin than the experimental point, then the calculated point is chosen as point on the outline or vice versa.

All SAS programs mentioned are available from the author on request.

REFERENCES

- [1] http://en.wikipedia.org/wiki/Balance_disorder
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