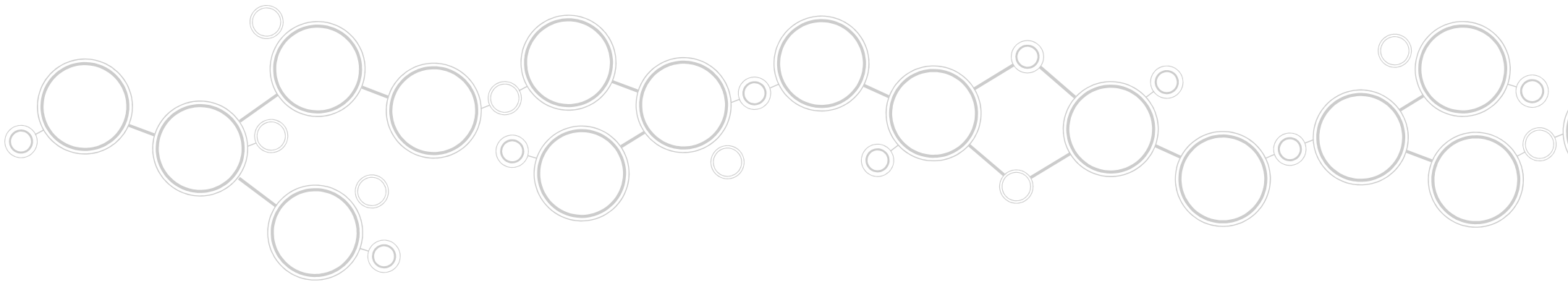




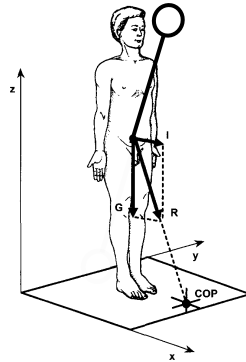
Different Methods of Calculating Body Sway Area

Thomas Wollseifen

Brighton (UK), 9th-12th October 2011



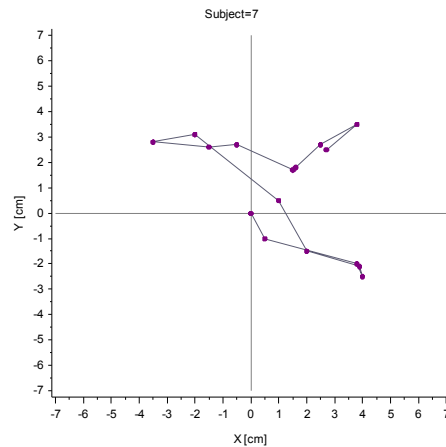
Posturography



Centre of pressure

Postural instability

- Parkinson's disease
- Sleeping problems
- Elderly subjects



Body sway path - trajectory

Body sway

- Length of path
- Velocity
- Distribution
- Body sway area

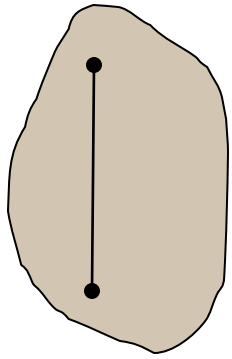
How to calculate the body sway area?

1) Area of convex hull

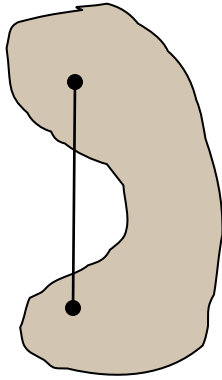
2) Area of ellipse –
Principal component analysis (PCA)

3) Mean of circle areas

1) Convex hull



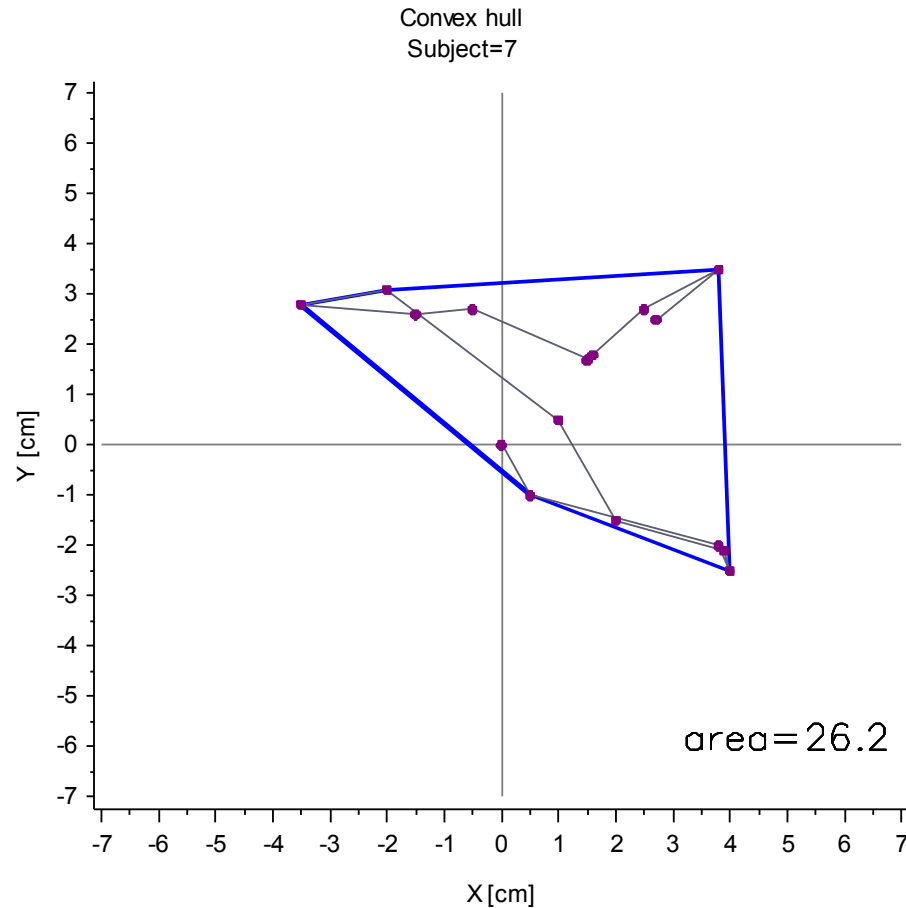
convex



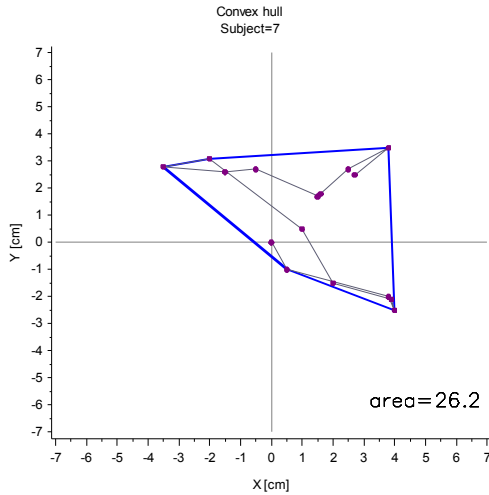
not convex

Different algorithms:

- Gift wrapping algorithm
- **Triangulation**
- etc.



Convex hull - Algorithm



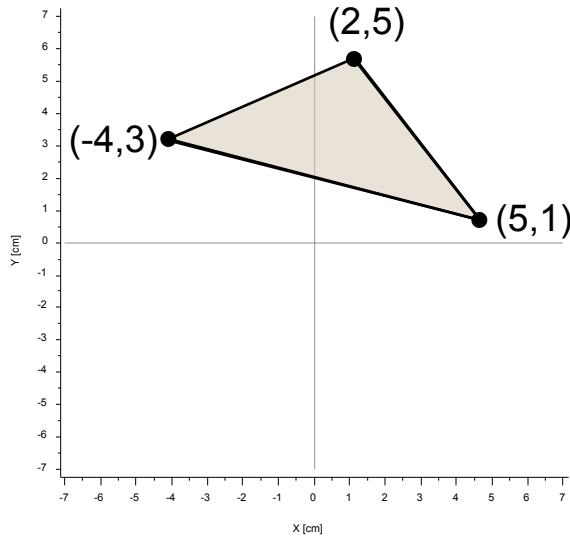
1) Triangulation of a point set (PROC G3GRID)

```
proc g3grid data=_ch00
    outtri=_ch01      → triangulation of data set
    out=_temp;
    grid &y.*&x.=&z.;
run;
```

2) Calculation of the determinant (of the convex hull = Area)

$$\text{Area}_{\text{Convex hull}} = \frac{1}{2} \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ \vdots & \vdots \\ x_n & y_n \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2} \left[(x_1 y_2 + x_2 y_3 + x_3 y_4 + \cdots + x_n y_1) - (y_1 x_2 + y_2 x_3 + y_3 x_4 + \cdots + y_n x_1) \right]$$

Find the area of this (convex) polygon → determinant



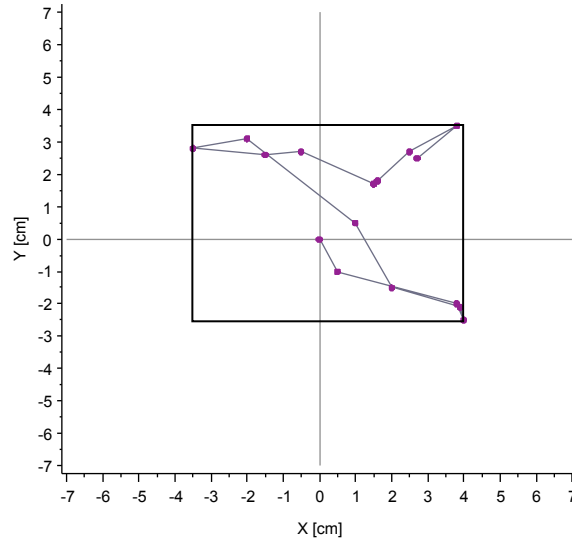
$$\text{Area} = \frac{1}{2} \begin{vmatrix} 2 & 5 \\ -4 & 3 \\ 5 & 1 \\ 2 & 5 \end{vmatrix} = \frac{1}{2} [(6 + (-4) + 25) - (-20 + 15 + 2)] = 15$$

```
data _det01;
  set &indata.;
  x2=lag(&x.);
  y2=lag(&y.);
  if not missing(x2) then d1=x2*&y.;
  else d1=0;
  if not missing(y2) then d2=y2*&x.;
  else d2=0;
run;
```

```
proc sql;
  create table _det02 as
    select *
      ,sum(d1) as sumd1
      ,sum(d2) as sumd2
      ,(calc. sumd2 - calc. sumd1)*0.5, as det
    from _det01
  ;
quit;
```

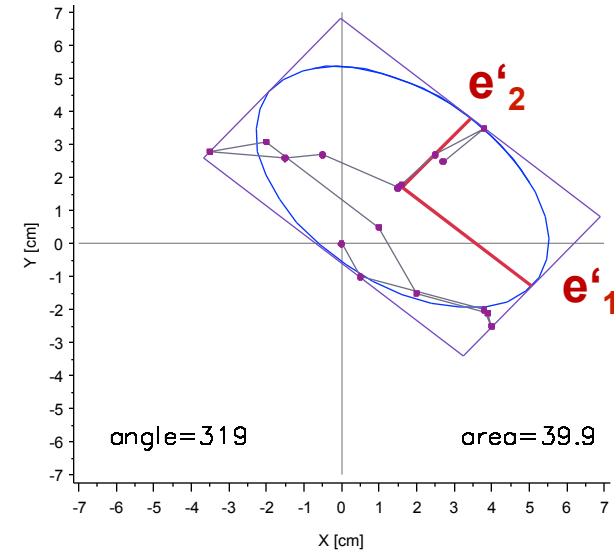
2) Principal component analysis (PCA)

$$A_{\text{PCA}} = \pi \cdot |e'_1| \cdot |e'_2|$$



Find a „better“
bounding box! →

PCA



Idea:

Reduce dimensionality of a point set by identifying the most significant directions (*Principal components = Eigenvectors*)

Principal component analysis with PROC PRINCOMP

```
proc princomp data=_pca01 COV OUT = _prins;  
  var &y. &x.;  
  ods output eigenvectors=_eigenvectors eigenvalues=_eigenvalues;  
run;
```

Covariance Matrix

	Y	X					
Y	4.526000000	-2.408333333	→ <table><tr><td>Cov(y,y)</td><td>Cov(y,x)</td></tr><tr><td>Cov(x,y)</td><td>Cov(x,x)</td></tr></table>	Cov(y,y)	Cov(y,x)	Cov(x,y)	Cov(x,x)
Cov(y,y)	Cov(y,x)						
Cov(x,y)	Cov(x,x)						
X	-2.408333333	5.195833333					

Eigenvalues of the Covariance Matrix

	Eigenvalue	Difference	Proportion	Cumulative
1	7.29242620	4.86301907	0.7501	0.7501
2	2.42940713		0.2499	1.0000

From covariance matrix to Eigenvectors / Eigenvalues

- Calculate 2x2 *covariances* cov(X,Y) of X,Y:

$$\text{cov}(X,Y) = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})$$

- Prepare *covariance matrix*:

Cov-Matrix			Y	X
Cov(y,y)	Cov(y,x)	Y	4.53	-2.41
Cov(x,y)	Cov(x,x)	X	-2.41	5.20

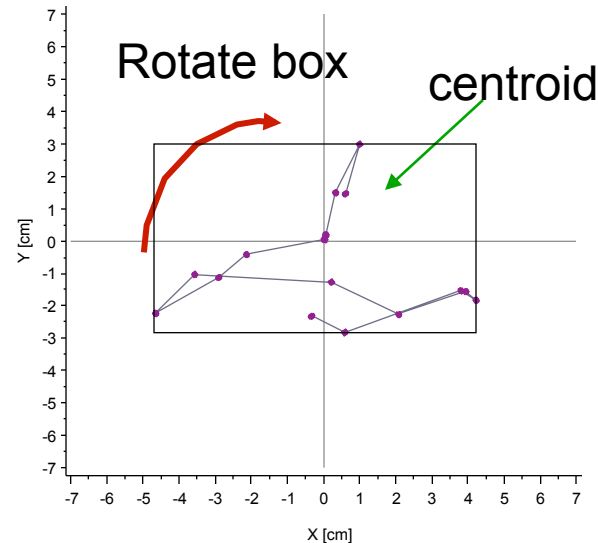
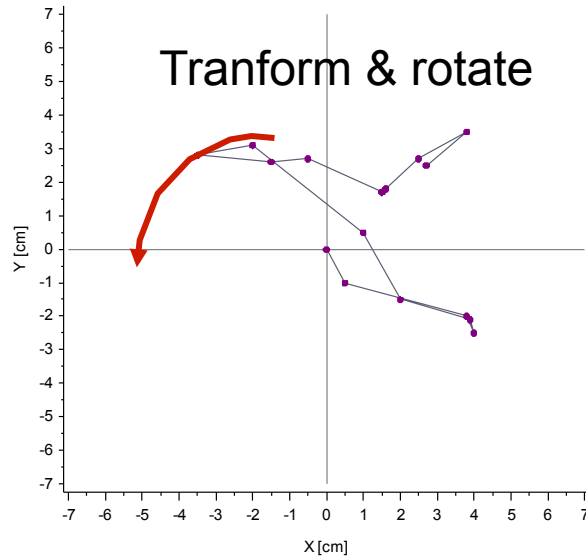
- Calculate *Eigenvalues* and *Eigenvectors* of the covariance matrix:

$$\det(A - \lambda E) = 0 \Rightarrow \begin{vmatrix} 4.53 - \lambda & -2.41 \\ -2.41 & 5.20 - \lambda \end{vmatrix} = 0$$

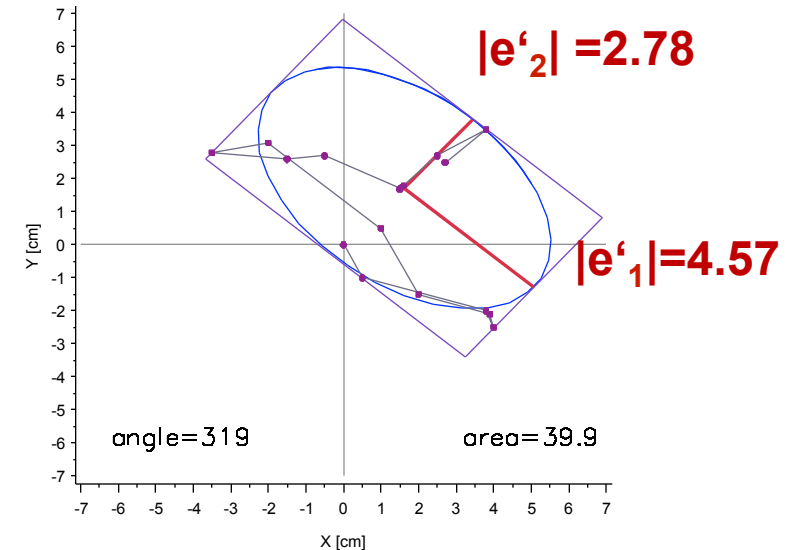
Eigenvalue ₁	7.29	Eigenvector e ₁	(0.75, -0.66)
Eigenvalue ₂	2.42	Eigenvector e ₂	(0.66, 0.75)

Find a bounding box - Algorithm

2. – 6.



7.



1. Calculate eigenvalues and eigenvectors e_1 and e_2 of covariance matrix (PROC PRINCOMP)

2. Calculate centroid of the point set

3. Translate point set such that the centroid is moved to the origin

4. Transform point set into polar coordinates (with angle θ and radius r)

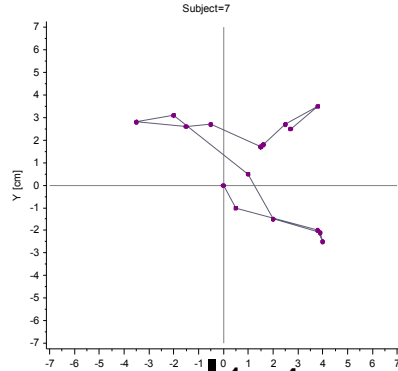
5. Rotate point set by angle of the 1st eigenvector

6. Calculate the bounding box and the length of the eigenvectors e'_1 and e'_2
Re-rotate bounding box by angle of the 1st eigenvector

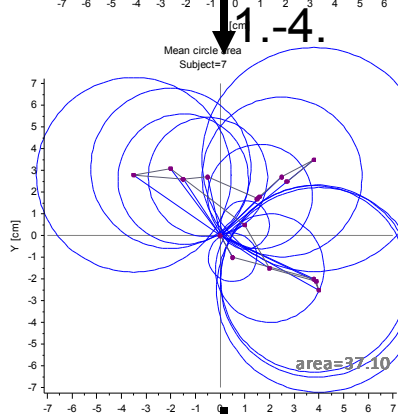
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7. $\text{Area}_{\text{PCA}} = \pi \cdot |e'_1| \cdot |e'_2|$

3) Mean circle area - Algorithm



1. Transformation into polar coordinates (r, θ)
2. Calculate the integer (INT) of each angle for each data point
3. Calculate max radius r for each angle (360 different angles possible)
4. For each remaining data point $p' (r, \theta)$ calculate the circle area
5. $A_m = \pi \cdot r^2$

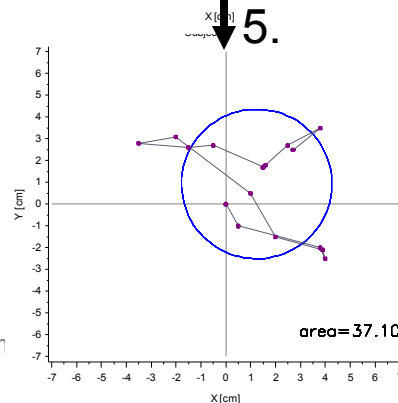


$$A_1 = \pi \cdot r^2$$

...

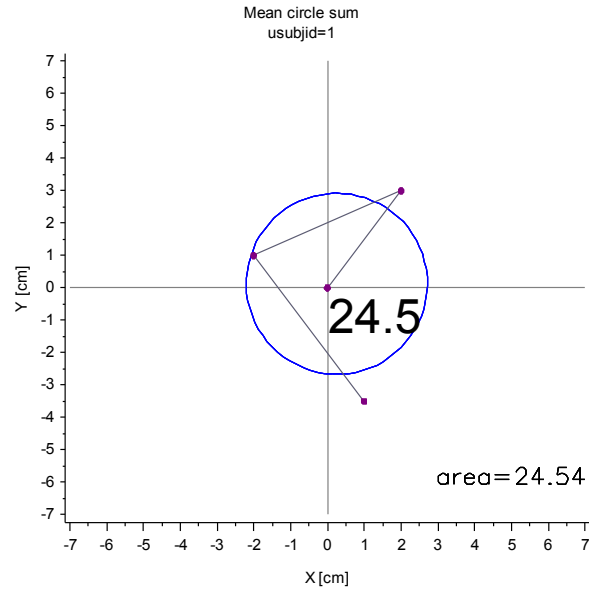
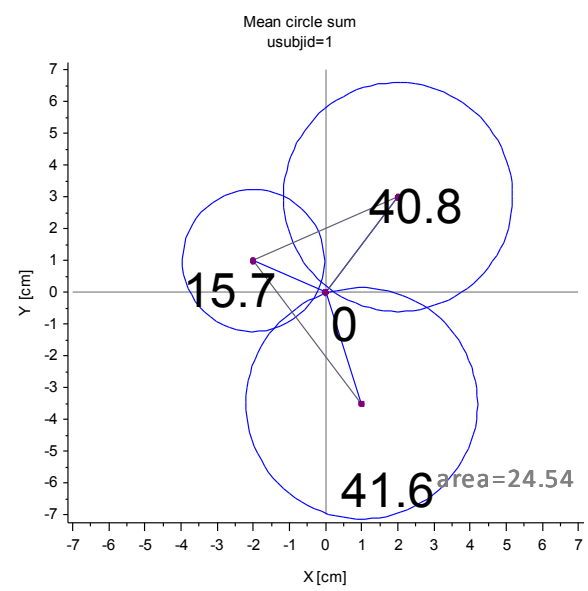
...

$$A_{14} = \pi \cdot r^2$$



$$A_m = \pi \cdot r^2$$

Mean circle area – Simple example



Polar
coordinates

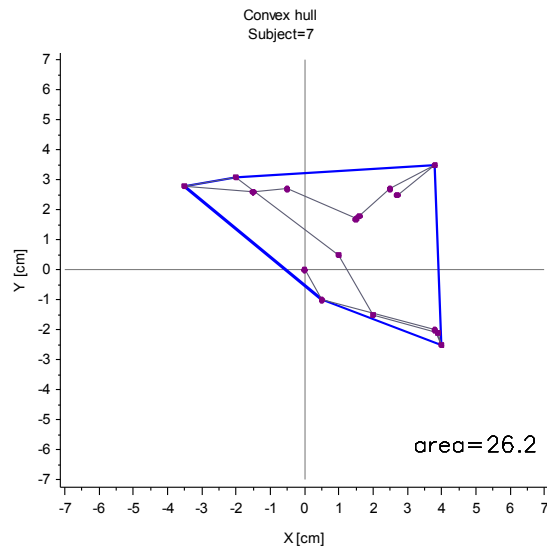
Circle Area

max_r	x	y	r	theta	pi	n	area_i	mean_area
0	0	0	0	0	3,141593	4	0	24,54
3,61	2	3	3,61	56	3,141593	4	40,84	24,54
2,24	-2	1	2,24	153	3,141593	4	15,71	24,54
3,64	1	-3,5	3,64	285	3,141593	4	41,63	24,54



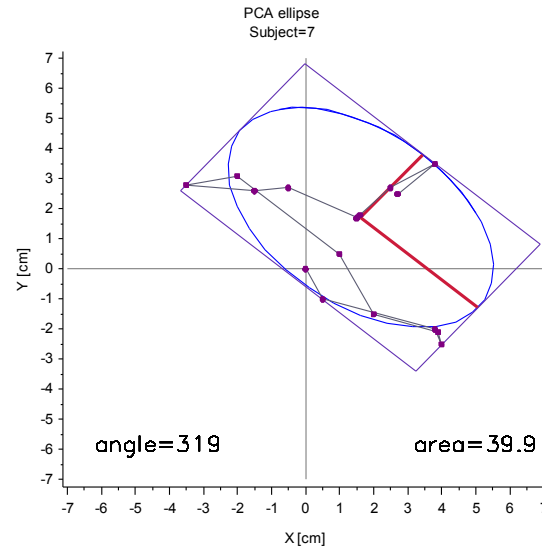
Comparison of methods

Convex hull



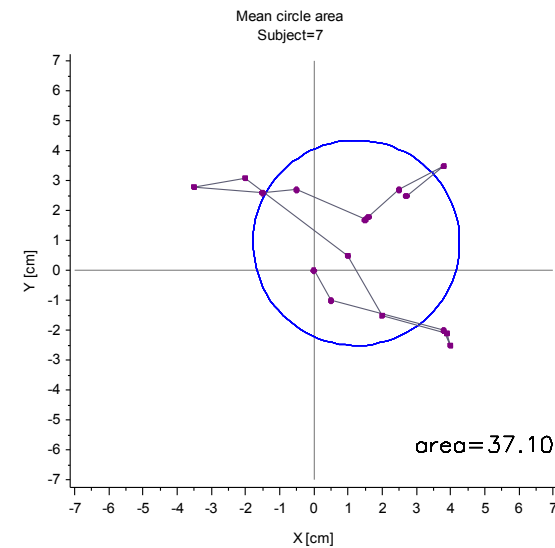
$$A_{CH}=26.2 \text{ [cm}^2\text{]}$$

PCA method



$$A_{PCA}=39.9 \text{ [cm}^2\text{]}$$

Circle method

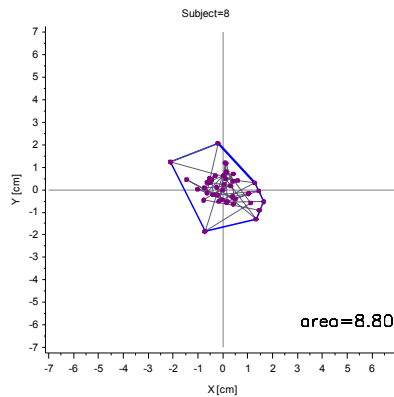


$$A_{CM}=37.1 \text{ [cm}^2\text{]}$$

Comparison of methods - different distributions

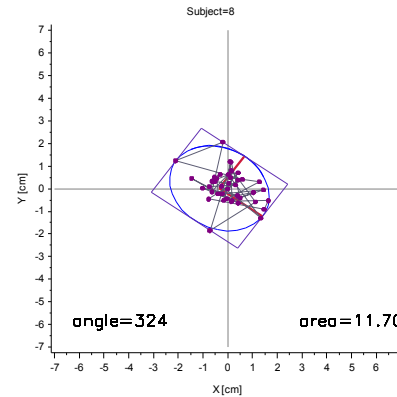
Normally distributed data

Convex hull



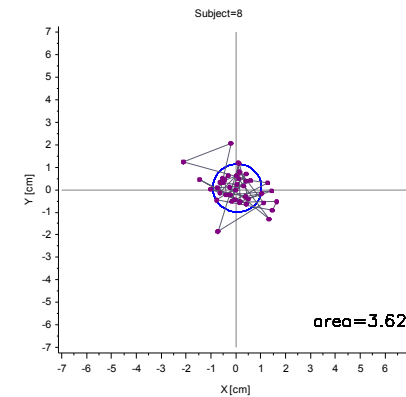
$$A_{CH}=8.8$$

PCA method



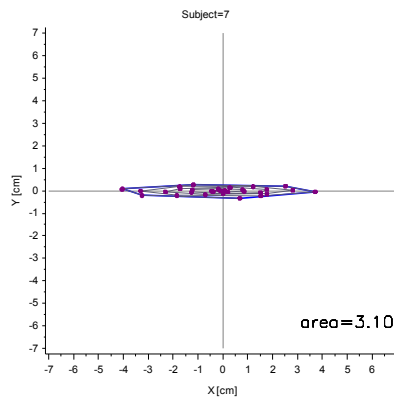
$$A_{PCA}=11.7$$

Circle method

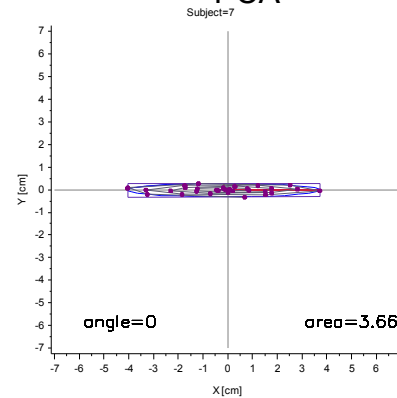


$$A_{CM}=3.6$$

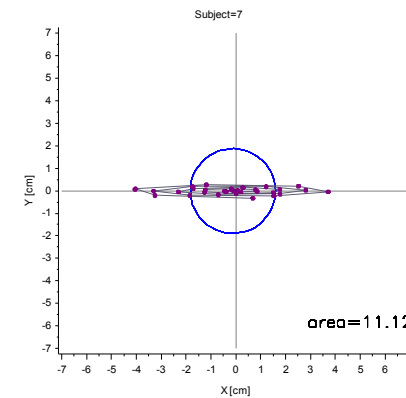
„Flat data“



$$A_{CH}=3.1$$



$$A_{PCA}=3.7$$



$$A_{CM}=11.1$$

Comparison of methods – different test data

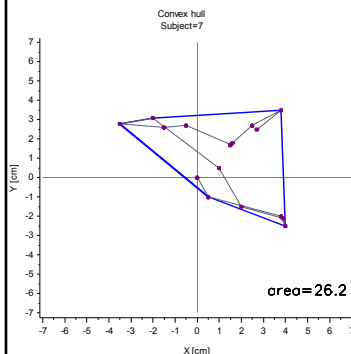
Test data	Calculation Method	N	Mean	Std	min	max	Pairwise comparison	p-value F-test	Normal Distr.?
test01	1 Convex hull	10	1.78	1.22	0.1	3.3	1-2: 0.9789 1-3:<0.0001 2-3:<0.0001	<0.0001	No
	2 PCA ellipse	10	2.08	1.44	0.1	3.9			
	3 Mean circle	10	13.88	5.65	11.1	29.6			
test02	1 Convex hull	10	3.58	2.45	0.3	6.7	1-2: 0.8785 1-3:<0.0001 2-3:<0.0001	<0.0001	Yes
	2 PCA ellipse	10	4.16	2.84	0.4	7.8			
	3 Mean circle	10	11.85	2.66	10.1	18.7			
test03	1 Convex hull	10	5.36	3.68	0.4	10.0	1-2: 0.8329 1-3: 0.0014 2-3: 0.0062	0.0010	No
	2 PCA ellipse	10	6.23	4.25	0.5	11.6			
	3 Mean circle	10	11.30	1.54	10.0	14.5			
test04	1 Convex hull	10	8.93	6.16	0.6	16.7	1-2: 0.8367 1-3: 0.6492 2-3: 0.9444	0.6683	Yes
	2 PCA ellipse	10	10.33	7.12	0.7	19.3			
	3 Mean circle	10	11.11	0.95	10.1	12.9			

Summary

Convex hull



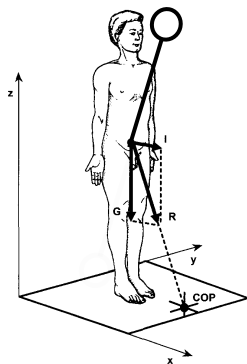
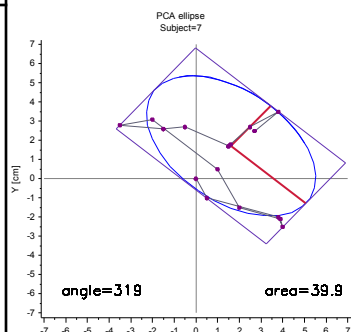
- $\text{Area}_{\text{convex hull}} < \text{Area}_{\text{PCA}}$
- Good approximation of the body sway area
- Data independent
- Encloses all data points



PCA ellipse



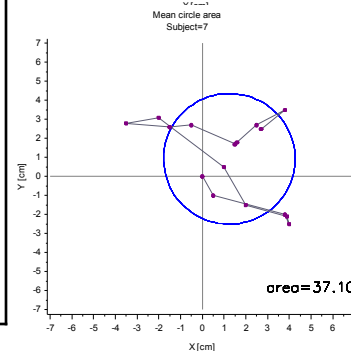
- Magnitude and direction
- Data independent
- Comparable result as convex hull
- Not all data points area enclosed



Circle method



- Fast algorithm
- Crude approximation of the sway area
- Depends on data distribution
- Not all data points area enclosed



Questions & Answers

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